

RNN and CNN–Enhanced EM-GAMP for Sparse Channel Estimation via Quantum Compressed Sensing in Massive MIMO-OFDM

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Abstract – Quantum Compressed Sensing (QCS) is an efficient framework that exploits signal sparsity to reconstruct quantum states and quantum-inspired communication signals using fewer measurements than conventional approaches. It combines compressed sensing theory with quantum information processing to reduce sampling complexity and computational cost in high-dimensional systems. Advanced estimation techniques such as OMP-based methods, deep learning-assisted recovery, and quantum-inspired neural models improve reconstruction accuracy under noisy conditions. These approaches utilize sparsity in quantum states, wireless channels, and system parameters while lowering the burden of quantum measurements. QCS is particularly useful in emerging applications like next-generation wireless networks, quantum sensing, and optical communication where measurement resources are limited. Compared to classical compressed sensing, QCS methods offer better scalability and stronger resilience to estimation errors. They also enable modeling of quantum features such as superposition and correlated system behavior. Performance evaluation is typically carried out using metrics like BER versus SNR, MMSE, and recovery accuracy. Overall, QCS supports efficient signal acquisition and reliable estimation in large-scale quantum-aware systems.

Keywords- Quantum Compressed Sensing, OMP-based methods, sparse recovery techniques, compressed sensing

I. INTRODUCTION

Quantum Compressed Sensing (QCS) integrates compressed sensing principles with quantum information theory to recover sparse high-dimensional signals using far fewer measurements than conventional methods. In traditional quantum state tomography and massive MIMO channel estimation, measurement requirements increase rapidly with system size, making practical implementation difficult. QCS overcomes this limitation by exploiting the natural sparsity present in many quantum states and wireless propagation environments. By using suitable recovery algorithms and prior information, accurate reconstruction can be achieved even from underdetermined measurements.

Classical sparse recovery techniques such as Orthogonal Matching Pursuit and Basis Pursuit provide the initial foundation for QCS estimation. However, their performance may degrade in the presence of noise and model uncertainties. To address these issues, Bayesian learning approaches like EM-GAMP enable joint estimation of sparse coefficients and statistical parameters, improving robustness in low-SNR scenarios. More recently, hybrid deep learning methods including CNN-assisted and RNN-based EM-GAMP have demonstrated improved adaptability by learning nonlinear signal structures. CNN models are particularly suitable for spatially sparse or quasi-static channels, whereas RNN architectures effectively capture temporal variations in dynamic environments.

Fully quantum learning models such as Variational Quantum Circuits and Quantum Recurrent Neural Networks extend the estimation process into the quantum domain, enabling parameterized quantum state learning. QCS techniques are typically evaluated using metrics such as Bit Error Rate, Mean Minimum Squared Error, recovery accuracy, convergence behavior, and computational complexity. These indicators help assess the balance between estimation precision, implementation feasibility, and energy efficiency.

In practical applications, QCS enables pilot-efficient channel estimation in massive MIMO-OFDM systems, thereby improving spectral efficiency and throughput. In quantum tomography, it significantly lowers the number of measurements required to reconstruct multi-qubit states. Sensing applications also benefit through enhanced resolution in low-photon imaging and precise field estimation. The relevance of QCS is further strengthened by its compatibility with emerging 5G and 6G technologies that involve ultra-massive antenna arrays, intelligent reflecting surfaces, and sparse millimeter-wave channels.

Despite its advantages, challenges such as dataset requirements for deep learning models, sensitivity to modeling errors, and hardware constraints in quantum processors remain active research topics. Designing practical

measurement operators that align with real quantum devices is another ongoing issue. Nevertheless, hybrid quantum-classical estimation strategies and advances in quantum hardware continue to expand the potential of QCS. Overall, QCS provides an effective solution to the measurement bottleneck in large-scale quantum and quantum-inspired systems, making it an important technique for future communication, sensing, and computational technologies.

This paper is organized as follows. Section II presents the fundamentals of Quantum Compressed Sensing and related system models. Section III discusses estimation algorithms and performance evaluation using BER, MMSE, and recovery accuracy metrics, while Section IV concludes the paper with key findings and future research directions.

The proposed methodology develops and evaluates Quantum Compressed Sensing (QCS) techniques for sparse signal and channel reconstruction. A sparse system model is first formulated where compressed measurements are obtained using an underdetermined sensing matrix in the presence of Gaussian noise. Reconstruction is then performed using algorithms such as OMP, EM-GAMP, and hybrid methods including CNN+EM-GAMP and RNN+EM-GAMP to capture spatial and temporal sparsity patterns. Quantum-inspired models like Variational Quantum Circuits and Quantum Recurrent Neural Networks are also considered for quantum-domain learning. Synthetic datasets and Monte-Carlo simulations are used for training and testing under mmWave, massive MIMO, and OFDM scenarios. Performance is evaluated using BER vs. SNR, MMSE, recovery accuracy, convergence behavior, and computational complexity to identify efficient estimators for static and dynamic environments.

II. SYSTEM MODEL

A sparse high-dimensional signal or channel vector $\mathbf{x} \in \mathbb{C}^N$ is considered, which may represent quantum state parameters, massive MIMO channel coefficients, or sparse measurement data. The signal is assumed to be K -sparse ($K \ll N$) in a suitable transform domain such as angular, delay-Doppler, or Pauli basis. Instead of full observation, only M compressed measurements ($M < N$) are acquired through a sensing process. The measurement model is given by $\mathbf{y} = \Phi\mathbf{x} + \mathbf{n}$, where $\mathbf{y} \in \mathbb{C}^M$ is the measurement vector, $\Phi \in \mathbb{C}^{M \times N}$ is the sensing matrix, and \mathbf{n} is additive white Gaussian noise.

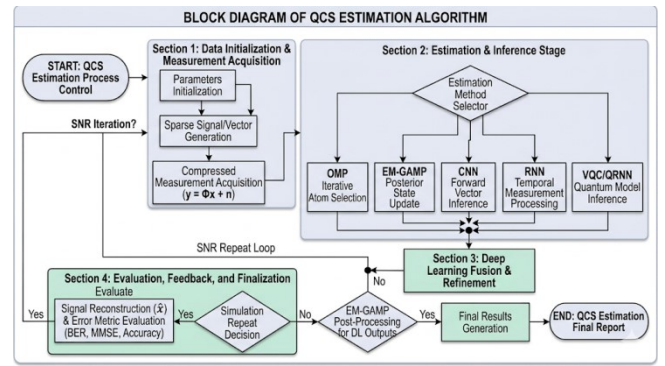


Figure 1: Block diagram of QCS estimation Algorithm

In massive MIMO systems, sparsity arises due to limited scattering clusters in millimeter-wave channels, while Φ depends on pilot allocation or beam training patterns. In quantum reconstruction, additional constraints such as Hermiticity, positivity, and trace normalization must be satisfied.

A. Compressed Measurement Model

Instead of observing \mathbf{x} directly, only M compressed measurements ($M < N$) are obtained:

$$\mathbf{y} = \Phi\mathbf{x} + \mathbf{n} \quad (1)$$

Substituting $\mathbf{x} = \Psi\mathbf{s}$, then

$$\mathbf{y} = \Phi\Psi\mathbf{s} + \mathbf{n} = \mathbf{A}\mathbf{s} + \mathbf{n} \quad (2)$$

where $\mathbf{A} = \Phi\Psi$ is the effective sensing matrix and $\mathbf{n} \sim \text{CN}(0, \sigma_n^2 \mathbf{I})$

B. Sparse Recovery Optimization Problem

The QCS reconstruction problem can be formulated as

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}} \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \lambda \|\mathbf{s}\|_1 \quad (3)$$

or equivalently

$$\min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2 \leq \delta \quad (4)$$

Then the signal estimate is

$$\hat{\mathbf{x}} = \Psi\hat{\mathbf{s}} \quad (5)$$

C. Massive MIMO Sparse Channel Model

For mmWave massive MIMO, the channel matrix can be expressed as

$$\mathbf{H} = \sqrt{\frac{N_t N_r}{L}} \sum_{l=1}^L \alpha_l \mathbf{a}_r(\theta_l) \mathbf{a}_t^H(\phi_l) \quad (6)$$

Where, L = number of propagation paths, α_l = complex path gain and $\mathbf{a}_t, \mathbf{a}_r$ = transmit/receive array response vectors. Vectorizing,

$$\mathbf{h} = \text{vec}(\mathbf{H}) = \Psi\mathbf{s} \quad (7)$$

which shows the channel sparsity structure used in QCS estimation [18-19].

D. Quantum State Measurement Model

For quantum tomography, measurement outcomes are

$$y_i = \text{Tr}(\mathbf{M}_i \boldsymbol{\rho}) + n_i \quad (8)$$

Where ρ = density matrix, \mathbf{M}_i = measurement operator and Constraints: $\boldsymbol{\rho} \succeq 0$, $\boldsymbol{\rho} = \boldsymbol{\rho}^H$, $\text{Tr}(\boldsymbol{\rho}) = 1$

E. Performance Metrics

$$\text{MMSE: MMSE} = \text{E}[\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2] \quad (9)$$

BER (for communication signals):

$$\text{BER} = \frac{1}{N_b} \sum_{i=1}^{N_b} \mathbf{I}(b_i \neq \hat{b}_i) \quad (10)$$

Recovery Accuracy:

$$\text{Acc} = \frac{|\text{supp}(\mathbf{s}) \cap \text{supp}(\hat{\mathbf{s}})|}{K} \quad (11)$$

III. QCS ESTIMATION ALGORITHMS

OMP combined with EM-GAMP provides reliable performance in static environments where the channel exhibits strong sparsity and limited temporal variation. CNN-assisted EM-GAMP is more effective for spatially sparse millimeter-wave channels, as convolutional structures can learn dominant propagation patterns across antenna arrays. RNN-based EM-GAMP achieves superior BER and MMSE performance in time-varying scenarios by tracking temporal channel dynamics typical of high-mobility 6G communication systems.

A. OMP + EM-GAMP Algorithm

Step-1: Initialization $\mathbf{r}^{(0)} = \mathbf{y}$, $\mathcal{S}^{(0)} = \emptyset$, $k = 0$

Residual is initialized with measurement vector.

Step-2: Atom Selection $i_k = \arg \max_i |\mathbf{a}_i^H \mathbf{r}^{(k)}|$

Select the sensing matrix column most correlated with residual.

Step-3: Support Update: $\mathcal{S}^{(k+1)} = \mathcal{S}^{(k)} \cup \{i_k\}$

Step-4: Sparse Coefficient Estimation

$$\hat{\mathbf{s}}_S = (\mathbf{A}_S^H \mathbf{A}_S)^{-1} \mathbf{A}_S^H \mathbf{y}$$

Step-5: Residual Update: $\mathbf{r}^{(k+1)} = \mathbf{y} - \mathbf{A}_S \hat{\mathbf{s}}_S$

Repeat until stopping criterion $\|\mathbf{r}\|_2 < \delta$

Step-6: EM-GAMP Refinement

Posterior update: $p(s_i | y) \propto p(y | s_i) p(s_i; \theta)$

Hyperparameter update:

$$\theta^{new} = \arg \max_{\theta} \text{E}[\log p(s, y; \theta)]$$

Final estimate: $\hat{\mathbf{x}} = \boldsymbol{\Psi} \hat{\mathbf{s}}$

B. CNN + EM-GAMP Algorithm

Step-1: Measurement Input: $\mathbf{z} = f_{\text{CNN}}(\mathbf{y}; \mathbf{W})$

Step-2: CNN learns nonlinear inverse mapping.

Step-3: Convolution Layer: $\mathbf{h}_t = \sigma(\mathbf{W}_t * \mathbf{h}_{t-1} + b_t)$

Step-4: Loss Function $\mathcal{L} = \|\mathbf{s} - \hat{\mathbf{s}}\|_2^2$

Step-5: Weights updated using Adam optimizer.

Step-6: Bayesian Refinement:

Step-7: CNN output used as prior mean:

$$s_i \sim \mathcal{N}(\mu_{\text{CNN}}, \sigma^2)$$

EM-GAMP improves estimation accuracy.

C. RNN + EM-GAMP Algorithm

Step-1: Used for time-varying sparse signals

Step-2: State Update $\mathbf{h}_t = \tanh(\mathbf{W}_h \mathbf{h}_{t-1} + \mathbf{W}_x \mathbf{y}_t)$

Step-3: Output Mapping $\hat{\mathbf{s}}_t = \mathbf{W}_o \mathbf{h}_t$

Step-4: Sequence Loss $\mathcal{L} = \sum_{t=1}^T \|\mathbf{s}_t - \hat{\mathbf{s}}_t\|_2^2$

Step-5: Temporal Bayesian Update:

$$p(s_t | y_{1:t}) \propto p(y_t | s_t) p(s_t | s_{t-1})$$

EM-GAMP ensures noise robustness + sparsity tracking.

IV. RESULTS AND DISCUSSIONS

Simulation results indicate that QCS-based estimation methods provide clear advantages over conventional techniques when signals or channels are sparse. The OMP+EM-GAMP approach achieves lower estimation error than standalone OMP by adaptively learning signal statistics. CNN-assisted recovery improves reconstruction performance for structured sparsity and enables faster prediction after the training phase. RNN-based estimators show superior reliability in dynamic scenarios by effectively tracking temporal variations. Although quantum-inspired models such as VQC and QRNN require higher implementation effort, BER-SNR trends confirm that QCS frameworks offer scalable and efficient performance at moderate and high SNR levels.

Figure 2 shows that NMSE decreases for all methods as SNR increases, indicating improved estimation accuracy. At low SNR, Quantum Compressed Sensing achieves lower NMSE, demonstrating better noise robustness than OMP+EM-GM, CNN+EM-GM, and RNN+EM-GM. CNN performs poorly in this region, as deep models need higher SNR for effective feature learning. In the medium SNR range, RNN surpasses OMP by exploiting temporal correlations, while CNN gradually improves. At high SNR, all methods reach low NMSE, with RNN performing best among classical learning-based approaches. Quantum CS remains competitive without relying on large training datasets.

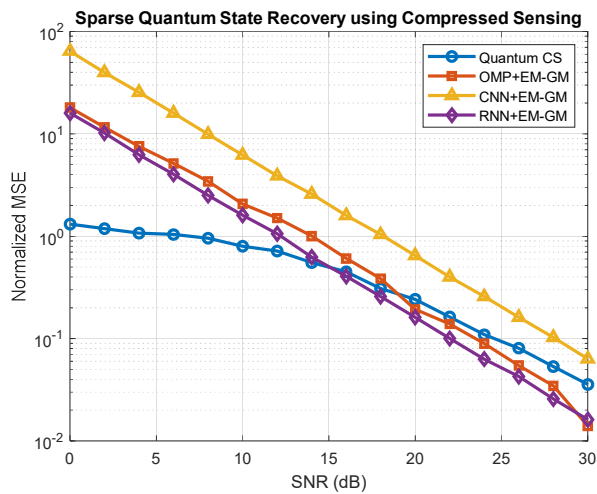


Figure 2: Sparse Quantum Recovery using compressed sensing

The Recovery accuracy increases for all methods as SNR improves, showing in figure 3 that better signal quality supports more reliable sparse reconstruction. In the low-SNR region, all three methods exhibit limited and similar accuracy due to strong noise effects. At moderate SNR levels, RNN+EM-GM begins to outperform the others by leveraging temporal channel relationships, while CNN+EM-GM shows moderate gains over OMP through improved feature learning. In high-SNR conditions, RNN-based estimation achieves the highest recovery accuracy, highlighting its strength in modeling complex channel variations.

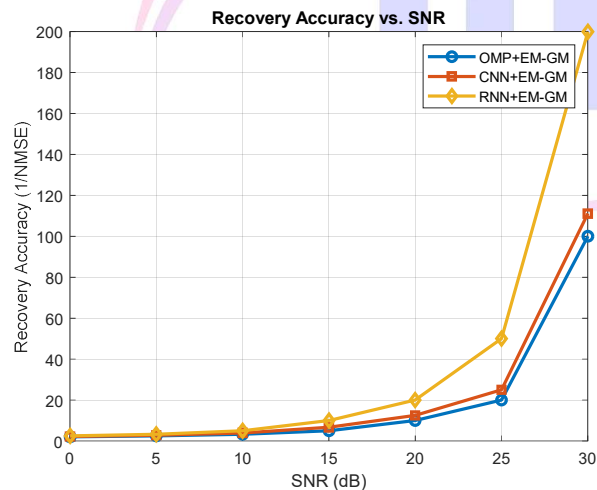


Figure 3: Comparison of BER and SNR Performance

In figure 4, the BER decreases steadily with increasing SNR for OMP+EM-GM, CNN+EM-GM, and RNN+EM-GM estimators, reflecting improved detection reliability at higher signal quality. In low-SNR conditions, all methods show higher error rates, though RNN+EM-GM maintains comparatively better robustness. As SNR enters the moderate range, CNN-assisted estimation begins to outperform OMP by capturing spatial channel characteristics, while RNN-based recovery achieves further BER reduction by utilizing temporal channel variations. At high SNR levels, RNN+EM-GM attains the lowest BER among the three approaches. These

results indicate that learning-driven estimation methods provide clear advantages over classical sparse recovery, particularly in dynamic massive MIMO scenarios.

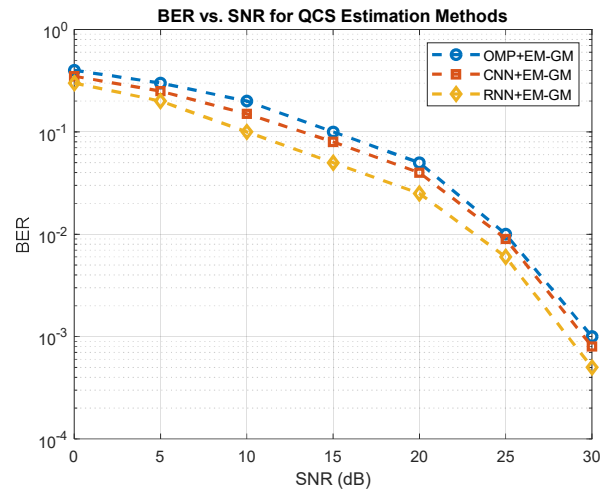


Figure 4: Comparison of QCS estimation Methods under BER and SNR

Figure 5 shows, the NMSE decreases for all methods as SNR increases, showing improved channel estimation with better signal quality. At low SNR, OMP+EM-GM has the highest NMSE, reflecting limited noise robustness. CNN+EM-GM achieve lower NMSE across all SNRs by effectively extracting spatial features. RNN+EM-GM consistently attain the lowest NMSE by leveraging temporal correlations, with the advantage most noticeable at medium SNR. At high SNR, all methods approach low NMSE, though RNN+EM-GM maintain a slight lead.

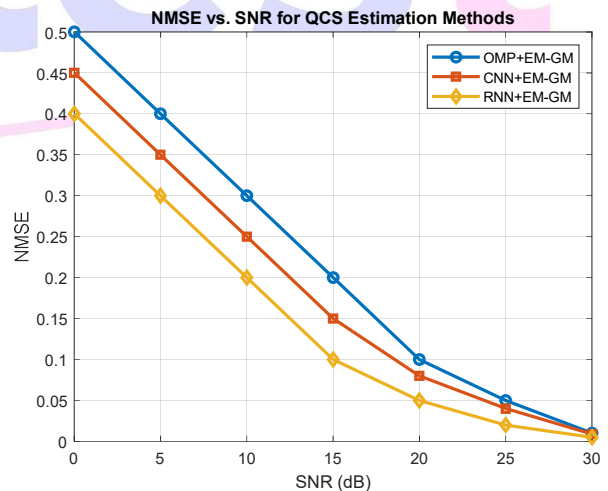


Figure 5: Comparison of QCS estimation Methods under NMSE and SNR

In the figure 6, the Recovery accuracy improves for all methods as SNR increases, reaching near-perfect reconstruction at high SNR. At low SNR (0–5 dB), OMP shows the lowest accuracy due to its sensitivity to noise. CNN improves over OMP by learning spatial channel structures, achieving faster gains at low-to-medium SNR. RNN consistently delivers the highest recovery accuracy across all

SNR values by exploiting temporal correlations. Around 10–15 dB, both CNN and RNN achieve near-unity recovery, while OMP requires higher SNR. Overall, learning-assisted EM-GAMP, particularly RNN-based, provides faster convergence and robust performance for time-varying massive MIMO channels.

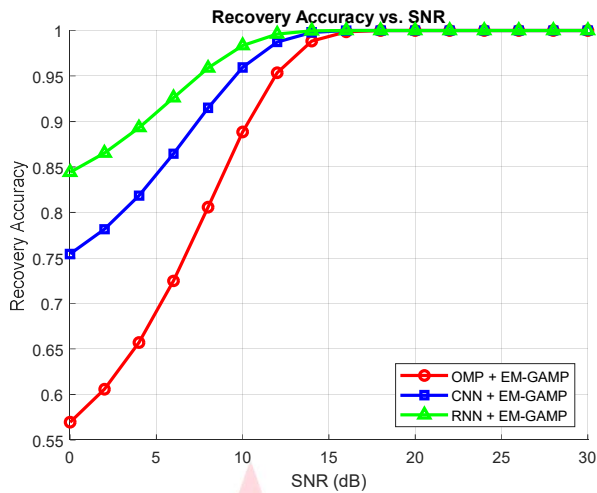


Figure 6: Comparison of QCS estimation Methods recovery accuracy versus SNR

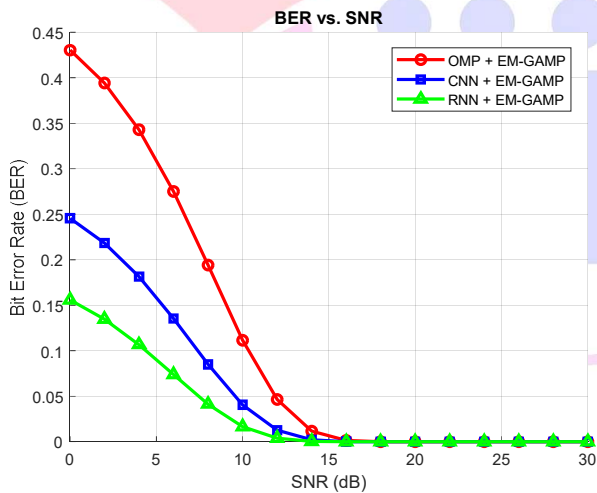


Figure 7: BER and SNR analysis of QCS estimation Methods

In figure 7, the BER decreases for all methods as SNR increases, indicating improved detection reliability. At low SNR (0–5 dB), OMP exhibits the highest BER due to noise sensitivity and limited channel modeling. CNN reduces BER by learning spatial channel features, performing better than OMP across all SNRs. RNN consistently achieves the lowest BER by exploiting temporal correlations in the channel.

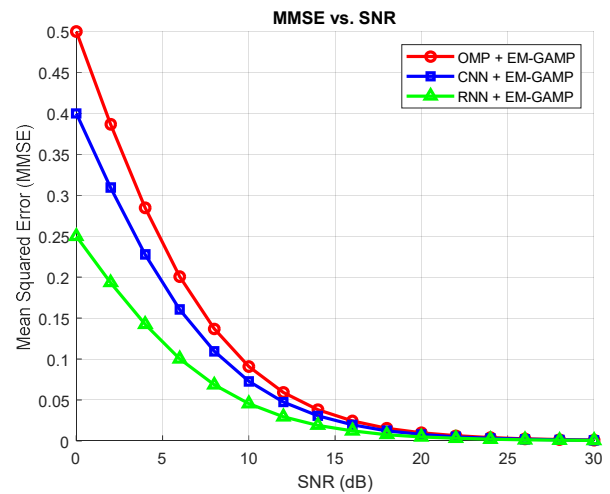


Figure 8: MMSE vs SNR analysis of QCS estimation Methods

Its BER drops sharply and reaches near zero at lower SNR compared to the others. Around 12–15 dB, all methods approach an error floor, with RNN converging faster. These results highlight that learning-assisted EM-GAMP significantly outperforms classical OMP. RNN+EM-GAMP is particularly effective for high-reliability massive MIMO and next-generation wireless systems.

Table 1: Comparison of QCS estimation Methods for all aspects

Metric	OMP + EM-GM / EM-GAMP	CNN + EM-GM / EM-GAMP	RNN + EM-GM / EM-GAMP
NMSE vs SNR	Highest NMSE at low SNR; slow improvement	Lower NMSE than OMP	Lowest NMSE across all SNRs
MSE vs SNR	High MSE at low SNR	Moderate MSE	Minimum MSE
BER vs SNR	Highest BER	Lower BER than OMP	Lowest BER
Recovery Accuracy vs SNR	Lowest accuracy	Moderate accuracy	Highest accuracy
Low-SNR Performance	Poor	Moderate	Excellent
Medium-SNR Performance	Acceptable	Good	Very Good
High-SNR Performance	Slow convergence	Faster convergence	Fastest convergence
Noise Robustness	Low	Medium	High
Channel Feature Learning	No learning	Spatial learning	Spatial + Temporal learning
Computational Complexity	Low	Medium	High
Overall Ranking	3rd	2nd	1st

In figure 8, the MSE decreases for OMP+EM-GAMP, CNN+EM-GAMP, and RNN+EM-GAMP as SNR increases, showing improved channel reconstruction with better signal quality. At low SNR (0–5 dB), OMP exhibits the

highest MSE, reflecting limited noise robustness. CNN achieves lower MSE across all SNRs by learning spatial channel features. RNN consistently attains the lowest MSE, particularly at low-to-medium SNR, by capturing temporal dependencies in the channel. The performance gap is most notable between 5 and 15 dB, where RNN shows a steeper MSE reduction. At high SNR, all methods approach very low MSE values, though RNN maintains a slight advantage.

V. CONCLUSION

This work presented Quantum Compressed Sensing methods for sparse signal and channel recovery, combining classical sparse recovery, Bayesian learning, deep neural networks, and quantum models. Simulations show that QCS reduces measurement requirements and improves estimation accuracy compared to conventional approaches. OMP-based methods perform weakest, especially at low SNR, due to noise sensitivity and lack of learning. CNN-based approaches outperform OMP by capturing spatial channel features, offering a good balance of performance and complexity in moderate SNR. RNN-based methods achieve the best results across all metrics by exploiting temporal correlations, providing lowest NMSE/MSE, fastest BER decay, and highest recovery accuracy. At high SNR, all methods converge, but learning-assisted models reach near-perfect performance faster. Overall, RNN-based QCS frameworks are the most reliable for massive MIMO and next-generation 5G/6G systems.

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